XXVII. *On the Nature of the Force producing the Motion of a Body exposed to Rays of Heat and Light.* *By* Arthur Schuster, *Ph.D., Demonstrator in the Physical Laboratory of Owens College. Communicated by* Balfour Stewart, *F.R.S.*

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Mr. Crookes has lately drawn attention to the mechanical action of a source of light on delicately suspended bodies *in vacuo*. I have made a few experiments which will, I think, throw some light on the cause of these phenomena, and assist us in the explanation of the manifold and striking experiments made by Mr. Crookes.

Whenever we observe a force tending to drive a body in a certain direction we are sure to find a force equal in amount acting in the opposite direction on the body or on the bodies from which the force emanates. It was with the view of finding the seat of this reaction that I have made the experiments described in these pages.

If the force is due directly to radiation, the reaction will be on the radiating body; if, on the other hand, it is due to any interior action, such as the one suggested by Professor Reynolds, the reaction will be on the exhausted vessel enclosing the bodies on which the force acts. I have been able to test this by experiment, and have found that the action and reaction are entirely between the light bodies suspended *in vacuo* and the exhausted vessel. The strength of the reaction is a measurable quantity, and hence we are able to calculate the absolute force acting on the bodies.

Description of Experiments.

The instrument best fitted for an experimental investigation of this kind is the one which has been called "Radiometer" by Mr. Crookes. These instruments have been made in great perfection by Mr. Geissler, of Bonn, under the name of "Light-mills." It is needless to describe the instrument in detail, as it does not materially differ from Mr. Crookes's radiometer. Light bodies are driven round continuously by the differential action of a source of light and heat on their faces, one of which is covered with lamp-black. The motion is in such a direction that the faces covered with lampblack recede from the light when they are turned towards it. The vessel containing the "light-mill" ends above and below in a vertical tube of about 1·5 centimetre diameter. A wire was laid round the tube near the top, and then brought over the top of the vessel in such a way that it formed a hook through which a cocoon-fibre could be drawn. A concave mirror was attached to this wire. The instrument was then suspended by a bifilar suspension from the top of a glass receiver which could be exhausted. It was found, however, that the amount of air in the receiver did not affect the experiments in any way. The lower end of the "light-mill," which was suspended in this way, was dipped into a small beaker, filled with oil, in order to steady the motion of the vessel and to bring it soon to its position of rest. The azimuth of the suspended vessel was read off on a scale by means of a dot of light concentrated by the concave mirror attached to the vessel. The time of vibration of the vessel was found to be about twenty-two seconds for a complete oscillation. The logarithmic decrement was found to be about 0·176.

The beam of light of an oxyhydrogen lamp was concentrated by means of a lens, and the vessel was placed at such a distance from the focus that the cone of light just enclosed the whole of the revolving mill. In this position the instrument showed the greatest sensitiveness. The "light-mill" revolved about 200 times in one minute.

The light was cut off at the beginning of the experiment by means of a screen, and the position of rest of the glass vessel was read off by means of the dot of light on the scale. The screen was then suddenly removed, and in every case a large deflection of the glass vessel containing the light-mill was observed. The vessel was deflected in a direction opposite to that in which the mill turned. When the velocity of the revolving mill had become constant, the outer vessel gradually came back to its original position of rest. This is a fact of great importance, and considerable care was taken to find out with what degree of accuracy it could be established. Owing to disturbances produced by various causes, the zero could never be determined within two or three divisions of the scale. The position of rest of the vessel when the mill was turning with constant velocity was always found to be within that distance from the original position of rest.

When the vessel containing the rotating "light-mill" had come to rest, the screen was suddenly replaced between the light and the mill. The mill now gradually came to rest, and at the same time the vessel was driven away in the opposite direction to that in which it moved on starting the experiment. The vessel, on stopping the light, was turned in the same direction in which the light-mill revolved. I shall give the readings of three experiments taken out of a great many made at different times and with different intensity of light.

	I.	II.	III.
Position of rest (light cut off)	0	0	0
Light suddenly turned on: successive elongations of the vessels containing the light-mill	+175	+88	+144
	− 95	−31	− 51
	+ 65	+17	+ 23
	− 43	− 4	− 11
	+ 29		+ 7
Position of rest (mill turning with constant velocity) .	0	+ 2	0
Light suddenly cut off: enlongations of vessel . .	−137	−62	−127
	+ 79	+28	+ 43
	− 47	− 8	− 20

Discussion of Experiments.

In order to understand fully the meaning of these experiments I shall take the most general case, and assume that forces act on the "light-mill" which are partly internal and partly external. I call internal forces all forces which act between the mill and its enclosure. We shall find that the experiments are not compatible with the existence of external forces.

Let X_1 be that part of the internal force and X_2 that part of the external force which is independent of the velocity of the "light-mill." It is found by experiment that, however small or large the whole force is, the mill always acquires a constant velocity. This velocity increases with the intensity of the force. In order to express this condition analytically, we must assume the existence of forces which increase as the velocity increases, and which always act in the opposite direction to X_1+X_2. I shall again take the most general case, and assume that these forces, which are functions of the velocity u, are partly internal and partly external. The internal force may be expressed by

$$-\varkappa_1 f(u).$$

That part of the external force which depends on the velocity may be expressed by

$$-\varkappa_2\phi(u).$$

The whole force which possibly can act on the light-mill can therefore always be expressed by

$$X_1+X_2-\varkappa_1 f(u)-\varkappa_2\phi(u).$$

The speed at which the mill will revolve uniformly will be determined by the equation

$$X_1+X_2-\varkappa_1 f(u)-\varkappa_2\phi(u)=0.$$

The force which acts on the enclosure is equal in amount, but opposite in direction, to the internal part of the whole force. (Any direct action of the light on the enclosure is left out of account.)

The force on the enclosure is therefore expressed by

$$-X_1+\varkappa_1 f(u).$$

When u has become equal to the greatest possible speed corresponding to the whole force, this expression is proved by experiment to be zero, because then the vessel returns to the original position of rest. Hence also

$$X_2-\varkappa_2\phi(u)=0,$$

which means that no external force acts on the "light-mill," as this equation is true for all intensities of light and therefore for all values of u.

All the Forces acting on the Light-mill are internal.

The accuracy of this statement naturally rests on the accuracy with which it can be ascertained that there is no force acting on the vessel when the speed of the mill has become uniform. It can be easily seen from what I have already said about the constancy of the position of rest, and from the deflection caused by the internal forces, that the existence of an external force equal to 5 per cent. of the internal force could not have escaped observation; and had this force been even less than half that amount, it would most likely have been detected. We therefore have at present no evidence whatever of any force directly referable to radiation.

The motion in the light-mill is wholly due to the forces acting between the revolving mill and its enclosure. It has not been the object of this investigation to find out what these forces are; and I must leave it therefore to Professor REYNOLDS to show in how far the experiments agree with his theory of the phenomenon. I ought, however, to mention that on suggesting the experiments described above to him, he at once told me how the reaction would make itself apparent according to his theory. His predictions have been fulfilled throughout. It was also through his courtesy that I was enabled to work with the neat instrument made by Mr. GEISSLER, and I have had his valuable aid throughout this investigation.

It is evident that the experiments agree in detail with the fact that two internal forces exist:—one, X_1, independent of the velocity; the other, $-\varkappa_1 f(u)$, increasing as the velocity increases and acting in a direction opposite to that of X_1. The force acting on the vessel is the reaction of the force which moves the mill, and is therefore expressed by

$$-X_1+\varkappa_1 f(u).$$

As long as $\varkappa_1 f(u)$, which vanishes with u, increases, it has not yet arrived at the value given by the equation

$$-X_1+\varkappa_1 f(u)=0.$$

In that case X_1 is greater than $\varkappa_1 f(u)$ and the vessel will be impelled in the negative direction, that is, in the direction opposite to that in which the mill revolves. When the velocity of the mill has become constant, the force acting on the vessel is zero, and the vessel will therefore return to its position of rest. When the light is suddenly removed, X_1 is suddenly removed, and the only force acting on the vessel is

$$\varkappa_1 f(u).$$

This force drives the vessel round in the opposite direction, that is, in the direction in which the mill is moving. As the velocity diminishes this force diminishes until it vanishes with the velocity.

The fact that, on suddenly removing the light, the deflection of the vessel was never found as large as when the light was turned on, is easily explained. The mill acquires the maximum speed in a shorter time than that required to reduce the velocity to zero when the light is removed. The force acting up to the time of the first elongation is smaller when the light is removed than when it is turned on, and hence the smaller elongation.

Calculation of the Intensity of the Force.

As it will be useful to those who are engaged in the explanation of these phenomena to know what is the strength of the force they have to account for, I have made an approximate calculation which will at any rate give an idea of the order of magnitude of the force. From the largest deflections observed on turning the light on, I calculate that the integral couple twisting the mill round a vertical axis is equal to the couple produced by the deflection of the dot of light through 125 scale divisions, owing to the bifilar suspension. The scale was at a distance of about 1100 scale divisions from the mirror. Considering that the dot of light moves through double the angle of deflection, I find that 0·06 is too great a value for the sine of the angle of deflection. The couple produced by such a deflection is (CLERK MAXWELL, 'Electricity and Magnetism')

$$L=\frac{ab}{4h}\,\omega\sin\alpha.$$

In this equation a and b are the distances of the upper and lower ends of the two fibres from each other, h is the vertical distance between the lines joining these upper and lower ends, ω is the weight of the suspended body, α the angle of deflection. In the experiments we have to give the following values to these constants (the units are those of the centimetre-gramme system):—

$$a=0{\cdot}25,$$
$$b=0{\cdot}08,$$
$$h=20,$$
$$\omega=32,$$
$$\sin\alpha=\ 0{\cdot}06\ ;$$
$$\text{hence } L=\ 0{\cdot}00048.$$

If p is the pressure on unit of area of the wings of the mill, A the area of these wings, and l the length of the arm of each wing,

$$L=lpA,$$
$$p=\frac{L}{lA}.$$

In Mr. GEISSLER'S light-mill, l is approximately 1·9 and $A=6{\cdot}45$ *; hence

$$lA=12{\cdot}2,$$
$$p=0{\cdot}000039.$$

The pressure on one square centimetre is therefore equal to the weight of the twenty-fifth part of a milligramme, or to 0·0006 grain.

The pressure on the wings of the light-mill was equal to that produced by the weight of a film of water equal in thickness to the length of a wave of blue light. This is the greatest pressure which I have been able to produce by means of the lime-light.

* This number is too large. See end of Appendix.

I have shown that an exterior force of 5 per cent. the amount of the interior force could have been detected. There cannot exist, therefore, an exterior force exerting a greater pressure on the unit of area of the light-mill than the 500th part of a milligram.

According to Prof. MAXWELL'S theory of light, an exterior force should exist; but this force would be smaller than the limits which we have given as having an appreciable effect on the experiments. By concentrating the rays of solar light in the ratio of one square foot to a square centimetre, however, the pressure on that square centimetre would be equal to 0·00004 gramme, a quantity which could not escape observation.

I think that the experiments described in the foregoing pages are interesting, not only by showing that the forces investigated by Mr. CROOKES are interior forces; but also by giving a method by means of which we are able to distinguish between such interior forces and forces directly referable to radiation. We may thus hope to find out whether such a pressure as that indicated by MAXWELL'S electromagnetic theory of light really exists.

APPENDIX (added Oct. 29, 1876).

It was suggested by Prof. MAXWELL to compare the motion of the glass vessel containing the radiometer with its equations of motion as deduced by him. I have therefore made a set of more careful experiments, and calculated from them, according to Prof. MAXWELL'S equations, the intensity of the force which acted on the vanes of the radiometer during my experiments. The number obtained has of course no other value than that of giving us an idea of the approximate magnitude of the force, as we have no accurate knowledge of the intensity of radiation actually falling on the vanes. My measurements show, however, that the motion of the vessel can be represented satisfactorily by a simple formula.

In order to deduce the equations of motion we shall, to simplify the calculation, make certain assumptions known to be only approximately correct. The first of them is, that the force acts with constant intensity during the experiments. The second is, that the resisting stress is proportional to the difference in the velocities of the radiometer and the vessel.

Let I and i be the moments of inertia of the glass vessel and of the light-mill.

Let y and x be their angular displacement.

Let $\varkappa$ be the coefficient of damping between the mill and the vessel.

Let K be the coefficient of damping between the enclosure of the light-mill and the outer vessel in which this enclosure is suspended.

Then if Hy is the force of restitution due to the bifilar suspension, the equations of motion are:—

1. For the mill

$$i\frac{d^2x}{dt^2}+\varkappa\left(\frac{dx}{dt}-\frac{dy}{dt}\right)=\mathrm{L}. \qquad (1)$$

2. For the enclosure

$$\mathrm{I}\frac{d^2y}{dt^2}+\varkappa\left(\frac{dy}{dt}-\frac{dx}{dt}\right)+\mathrm{K}\frac{dy}{dt}+\mathrm{H}y=-\mathrm{L}. \quad \ldots \quad (2)$$

These equations can be easily integrated, yet it would be difficult to determine the constants occurring in the final equation. If we consider, however, that i is very small compared to I, and that, therefore, the velocity produced by the force L on the mill is very great compared to that produced on the vessel, we can in equation (1) neglect $\frac{dy}{dt}$ in comparison with $\frac{dx}{dt}$, and we then get as integral

$$\frac{dx}{dt}=\frac{\mathrm{L}}{\varkappa}\left(1-e^{-\frac{\kappa}{i}t}\right). \quad \ldots \quad (3)$$

This equation means that the velocity of the mill gradually increases until it reaches the final velocity $\frac{\mathrm{L}}{\varkappa}$. Putting the value of $\frac{dx}{dt}$ from (3) into (2), we have

$$\mathrm{I}\frac{d^2y}{dt^2}+(\mathrm{K}+\varkappa)\frac{dy}{dt}+\mathrm{H}y=-\mathrm{L}e^{-\frac{\kappa}{i}t} \quad \ldots \quad (4)$$

The solution of this equation is

$$y=\mathrm{A}e^{-\lambda t}\sin(nt+\alpha)+\mathrm{B}e^{-\frac{\kappa}{i}t}, \quad \ldots \quad (5)$$

with the conditions

$$\mathrm{I}(\lambda^2-n^2)-(\mathrm{K}+\varkappa)\lambda+\mathrm{H}=0 \quad \ldots \quad (6)$$

$$2\mathrm{I}\lambda-(\mathrm{K}+\varkappa)=0 \quad \ldots \quad (7)$$

$$\mathrm{B}\left(\mathrm{I}\frac{\varkappa^2}{i^2}-(\mathrm{K}+\varkappa)\frac{\varkappa}{i}+\mathrm{H}\right)=-\mathrm{L}; \quad \ldots \quad (8)$$

from (6) and (7) we can determine $\mathrm{K}+\varkappa$ and I, as λ and n are found by observation, and H is given by the data of the bifilar suspension. In order to determine L from equation (8) we must find $\frac{\kappa}{i}$ and B.

For $t=0$ we have $y=0$ and $\frac{dy}{dt}=0$.

This gives the boundary conditions,

$$\mathrm{A}\sin\alpha+\mathrm{B}=0 \quad \ldots \quad (9)$$

$$\mathrm{A}(\lambda\sin\alpha-n\cos\alpha)+\mathrm{B}\frac{\kappa}{i}=0. \quad \ldots \quad (10)$$

Eliminating A and B out of these equations, we get

$$\lambda\sin\alpha-n\cos\alpha-\frac{\kappa}{i}\sin\alpha=0. \quad \ldots \quad (11)$$

It is found by experiment that after a few swings the vessel vibrates round its position of rest. The motion then is given by

$$y=\mathrm{A}e^{-\lambda t}\sin(nt+\alpha).$$

If the time t for the elongations is only given approximately, the elongations will determine A.

For the first elongation y_1 we have, if the time is t_1,

$$y_1 = \mathrm{A}\left[e^{-\lambda t_1}\sin(nt_1+\alpha) - \sin\alpha e^{-\frac{\kappa}{i}t_1}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

In this equation $-\mathrm{A}\sin\alpha$ is put for B.

Differentiating (5) we have for the first elongation, as the velocity is zero,

$$0 = e^{-\lambda t_1}\{n\cos(nt_1+\alpha) - \lambda\sin(nt_1+\alpha)\} + \sin\alpha\frac{\kappa}{i}e^{-\frac{\kappa}{i}t_1}. \quad . \quad . \quad . \quad . \qquad (13)$$

The equations (12) and (13) will determine t_1 and α. They can, however, be brought into a more convenient form.

Eliminating $e^{-\frac{\kappa}{i}t}$ out of (12) and (13), and introducing a new variable β, such that $\tan\beta = \frac{n}{\lambda}$, we get

$$\sin(\alpha-\beta) = \frac{-n\mathrm{A}}{y_1\sqrt{n^2+\lambda^2}}\sin nt_1\, e^{-\lambda t_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

We also get easily

$$\cos(\alpha-\beta) = \frac{n\mathrm{A}}{y_1\sqrt{n^2+\lambda^2}}\left(\cos nt_1\, e^{-\lambda t_1} + e^{-\frac{\kappa}{i}t_1}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

Knowing α, we shall be able to determine $\frac{\kappa}{i}$ from (11), and hence to calculate the force L.

In the experiments actually made, 20 successive elongations were observed. It was found that after the third the vessel vibrated round its position of rest. All the elongations after the third were therefore used to determine the time of vibration and the logarithmic decrement. Another set of observations was taken after the light had been removed, and the position of rest was determined in this way for the case in which no light falls on the mill.

In the bifilar suspension the distance between the two threads at the top was 0·25, between the two threads at the bottom 0·08. The vertical distance from the top to the bottom of the suspension was 19·6, and the weight of the vessel was 31·2. The units used are those of the centimetre-gramme system. We calculate from these data

$$\mathrm{H} = 0{\cdot}00796.$$

We find by experiment the time of half a vibration 11·03, and the logarithmic decrement 0·02978. Hence we calculate

$$\lambda = 0{\cdot}006219,$$

$$n = 0{\cdot}28482.$$

Equations (6) and (7) give

$$\mathrm{H} = \mathrm{I}(\lambda^2 + n^2).$$

Putting the value, and remembering that $K+\varkappa=2I\lambda$, we find

$$\left.\begin{aligned} L&=B\left(\left(\frac{\varkappa}{i}-\lambda\right)^2+n^2\right) \\ &=BH\frac{n^2}{(\lambda^2+n^2)}\frac{1}{\sin^2\alpha} \end{aligned}\right\} \qquad (16)$$

We now turn to the determination of B from the successive elongations of the vessel, and from the approximate time which was observed we find $A=99\cdot5$ in divisions of the scale. The mean of two experiments gave for the first elongation 124·3. Hence we calculate from (14) and (15)

$$t_1=9\cdot15$$

and

$$-\alpha=68^\circ\ 33'.$$

From this we get $B=92\cdot6$ in divisions of the scale. Reducing B to angular measure, we find

$$B=0\cdot02953.$$

The distance of the scale from the mirror was 1526 divisions of the scale. In the calculation of B we had to apply a correction, because the scale did not stand parallel to the mirror when the mirror was at rest. This was done by multiplying B with 0·9771. (This number was obtained by measuring the angle of inclination of the scale.) Putting this value of B into (16), we get

$$L=0\cdot0002714.$$

From (10) we get $\frac{\kappa}{i}=0\cdot11795$; and we have therefore now determined all the constants of the equation. In order to see in how far the values obtained correctly represent the experiments, the second elongation was calculated by the formula to be 172·3 divisions of the scale. By experiments it was found to be 171. As the second elongation did not enter into any determination of the constants, the agreement seems satisfactory.

Calculating the position of rest from the successive elongations when the mill was turning, and afterwards when the mill was at rest, a difference of 1·3 division of the scale was noticed. The vessel appeared to be deflected 1·3 division of the scale in a direction opposite to that in which the mill was turning. Considering the difficulty which besets the exact determination of the zero-point when the light is turned on the mill, owing to changes in the intensity of the light, external air-currents, &c., we may regard the two positions of rest to be, as far as our experiment could determine them, identical. The permanent deflection either way, if it existed, must, we may say with certainty, have been smaller than 5 divisions of the scale.

The moment of any external force, if existing, must therefore have been smaller than 0·000013, or less than 5 per cent. of the internal force.

In order to get a numerical value for the pressure on the unit of area of the wings of the mill, we must divide the couple L by the length of the arms and the area of the wings. The length of the area was approximately 1·9 and the area 4·84; hence the pressure on a square centimetre during the experiments was

$$0{\cdot}000030 \text{ grammes,}$$

which is equal to the weight of a layer of water covering the surface, and equal in thickness to the length of a wave of ultra-violet light. The pressure is about double that determined by Mr. CROOKES in one of his experiments*. Considering the greater intensity of the light used in my experiments, the results agree pretty well with each other.

In the original paper the area of the wings was erroneously given as 6·45; the pressure was therefore found too small in the proportion of 4·84 to 6·45. The pressure deduced by the rough method of the original paper should therefore have been

$$p=0{\cdot}000052.$$

The number only professed to be an outer limit to the pressure, the real pressure being necessarily smaller. The intensity of the light used in the more careful experiments described in these pages was decidedly smaller than that used in the original experiments. Under these circumstances I think the agreement between the two numbers is as close as could be expected.

* Lecture delivered at the Royal Institution, February 11; reprinted, 'Quarterly Journal of Science,' April 1876.